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12. a)

Figure Number, n	Number of Toothpicks, t
1	3
2	5
3	7
4	9

As the figure number increases by 1, the number of toothpicks increases by 2.

b) Each figure needs 1 toothpick for the left edge, plus 2 additional toothpicks for each triangle in the figure. That is, number of toothpicks = $1 + (\text{figure number}) \times 2$. An expression is: $1 + 2n$, or $2n + 1$

c) Substitute $n = 45$ in the expression $2n + 1$.

$$2(45) + 1 = 90 + 1 \\ = 91$$

There are 91 toothpicks in figure 45

d) An equation is: $t = 2n + 1$

e) Substitute $t = 17$ in the equation $t = 2n + 1$, then solve for n .

$$17 = 2n + 1 \quad \text{Subtract 1 from each side.}$$

$$17 - 1 = 2n + 1 - 1$$

$$16 = 2n$$

Divide both sides by 2.

$$\frac{16}{2} = \frac{2n}{2}$$

$$8 = n$$

Figure 8 has 17 toothpicks.

I could check my answer by substituting $t = 17$ and $n = 8$ in the equation $t = 2n + 1$.

$$\text{Left side} = 17 \quad \text{Right side} = 2(8) + 1 \\ = 17$$

Since the left side equals the right side, the answer is correct.

13. a)

Number of Tables, n	Number of People, p
1	6
2	10
3	14
4	18

b) As the number of tables increases by 1, the number of people who can be seated increases by 4.

c) I can develop a pattern.

Since the number of people increases by 4 each time, I know I have to multiply the number of tables by 4 and then find out what I need to add or subtract to get the number of people.

From the table, when $n = 1$, $p = 6 = 4(1) + 2$, so I have to add 2 to get the number of people.

I can generalize this pattern by writing an expression or an equation.

- d) When the number of people who can be seated is represented by p and the number of tables by n , then an equation is: $p = 4n + 2$.

To verify this equation, I substitute a pair of values of p and n in the equation to check that the left side is equal to the right side. I choose $n = 3$ and $p = 14$.

Substitute $n = 3$ and $p = 14$ in the equation $p = 4n + 2$:

$$\begin{array}{ll} \text{Left side} = 14 & \text{Right side} = 4(3) + 2 \\ & = 14 \end{array}$$

Since the left side is equal to the right side, the equation is correct

- e) Substitute $p = 41$ in the equation $p = 4n + 2$ and solve for n .

$$41 = 4n + 2 \quad \text{Subtract 2 from each side.}$$

$$41 - 2 = 4n + 2 - 2$$

$$39 = 4n$$

Divide both sides by 4.

$$\frac{39}{4} = \frac{4n}{4}$$

$$9.75 = n$$

Since the number of tables cannot be a decimal, I would need 10 tables to seat 41 people.

I can check the answer by substituting $n = 10$ in the equation.

$$\begin{array}{l} p = 4(10) + 2 \\ = 42 \end{array}$$

This means 41 people can be seated at 10 tables and my answer is correct.

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15. a)

Number of Toppings, n	Cost, C (\$)
0	9.00
1	9.75
2	10.50
3	11.25
4	12.00
5	12.75

- b) The cost of a pizza, C , is \$9.00 plus \$0.75 \times (number of toppings, n).
So, an equation is $C = 9.00 + 0.75n$.

To verify the equation, substitute $n = 3$ in the equation $C = 9.00 + 0.75n$.

$$\begin{array}{l} C = 9.00 + 0.75(3) \\ = 9.00 + 2.25 \\ = 11.25 \end{array}$$

From the table, $C = 11.25$ when $n = 3$, so the equation is correct.

- c) I used the equation. Substitute $C = 15.00$, then solve for n .

$$15.00 = 9.00 + 0.75n$$

Subtract 9.00 from each side.

$$15.00 - 9.00 = 9.00 + 0.75n - 9.00$$

$$6.00 = 0.75n$$

Divide both sides by 0.75.

$$\frac{6.00}{0.75} = \frac{0.75n}{0.75}$$

$$8 = n$$

The pizza has 8 toppings.

From the table, when the number of toppings increased from 0 to 4, the cost increased from \$9 to \$12, an increase of \$3. The cost should increase another \$3 to \$15 when the number of toppings increases from 4 to 8 toppings.

16. a) The cost is \$12 plus \$1.50 \times (number of windows cleaned). Let C represent the total cost in dollars and let w represent the number of windows cleaned, then an equation is $C = 12 + 1.50w$.

Substitute $w = 1$ into the equation:

$$C = 12 + 1.50(1)$$

$$= 13.50$$

\$13.50 is the cost to clean one window; so the equation is correct.

- b) Substitute $C = 28.50$ into the equation, then solve for w .

$$28.50 = 12 + 1.50w$$

$$28.50 - 12 = 12 + 1.50w - 12$$

$$16.50 = 1.50w$$

$$\frac{16.50}{1.50} = \frac{1.50w}{1.50}$$

$$11 = w$$

Clint cleaned 11 windows.

I can verify this by substituting $w = 11$ into the equation.

$$C = 1.50w + 12$$

$$= 1.50(11) + 12$$

$$= 16.50 + 12$$

$$= 28.50$$

Since this matches the given value of C , the answer is correct.

17. Let p represent the number of patio stones needed and let n represent the size number of the garden. Create a table.

Garden Size, n	Number of Patio Stones, p
1	$8 = 2(1) + 6$
2	$10 = 2(2) + 6$
3	$12 = 2(3) + 6$
4	$14 = 2(4) + 6$

As the garden size number increases by 1, the number of patio stones increases by 2. When the size number is 1, 8 patio stones are needed. So, an equation is $p = 2n + 6$.

To determine the garden size of a patio with 152 stones, substitute $p = 152$ into the equation $p = 2n + 6$, then solve for n .

$$152 = 2n + 6 \quad \text{Subtract 6 from each side.}$$

$$152 - 6 = 2n + 6 - 6$$

$$146 = 2n \quad \text{Divide both sides by 2.}$$

$$\frac{146}{2} = \frac{2n}{2}$$

$$73 = n$$

The garden size is 73.

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20. a) As t increases by 1, v decreases by 4. Since $v = 80$ when $t = 1$, an equation is: $v = 84 - 4t$

b) Substitute $t = 3$ and $v = 72$ in the equation.

Left side: 72

$$\begin{aligned} \text{Right side: } 84 - 4(3) &= 84 - 12 \\ &= 72 \end{aligned}$$

Since the left side is equal to the right side, the equation is correct.

21. a)

Number of cuts	1	2	3	4	5	6	7	8	9	10
Number of pieces	2	4	8	16	32	64	128	256	512	1024

b) As the number of cuts increases by 1, the number of pieces doubles.

The number of pieces are powers of 2.

c) To determine the number of pieces, double the number of pieces after 10 cuts 5 times:

$$1024 \times 2 \times 2 \times 2 \times 2 \times 2 = 32\,768$$

There will be 32 768 pieces after 15 cuts.

d) Rewrite the number of pieces as powers of 2:

Number of cuts	1	2	3	4	5
Number of pieces	2	$(2) \times 2 = 2^2 = 4$	$(2 \times 2) \times 2 = 2^3 = 8$	$(2 \times 2 \times 2) \times 2 = 2^4 = 16$	$(2 \times 2 \times 2) \times 2 = 2^5 = 32$

Let P represent the number of pieces and let n represent the number of cuts. Since the number of pieces are powers of 2, P is related to n by the equation: $P = 2^n$

e) 15 cuts results in 32 768 pieces. I know that 16 cuts will result in double this number of pieces, or 65 536 pieces. So, 16 cuts have to be made to get more than 50 000 pieces.